A Dynamic Programming Approach for Optimal Liner Containership Fleet Planning

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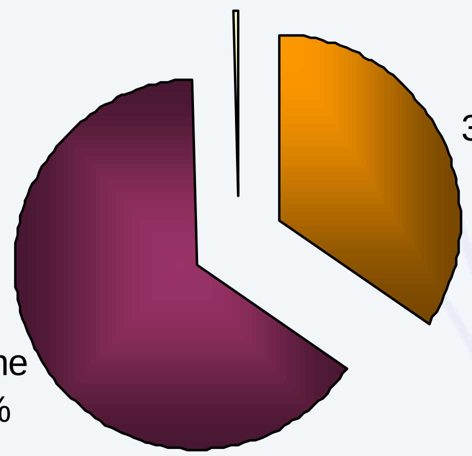
Contents

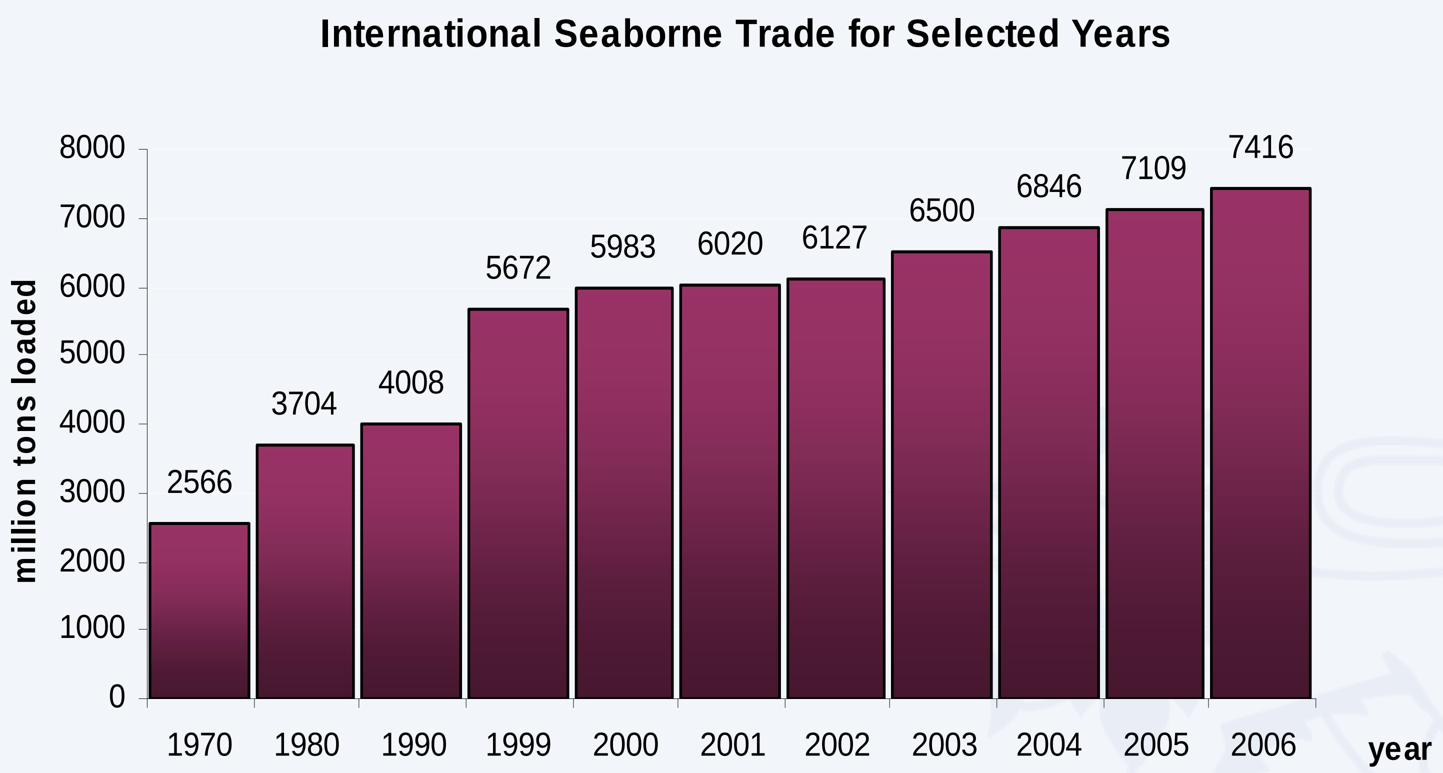
Introduction-Seaborne(1/2)

World Trade By Transport Modes



Source: Kumar & Hoffmann (2002)

Introduction-Seaborne(2/2)

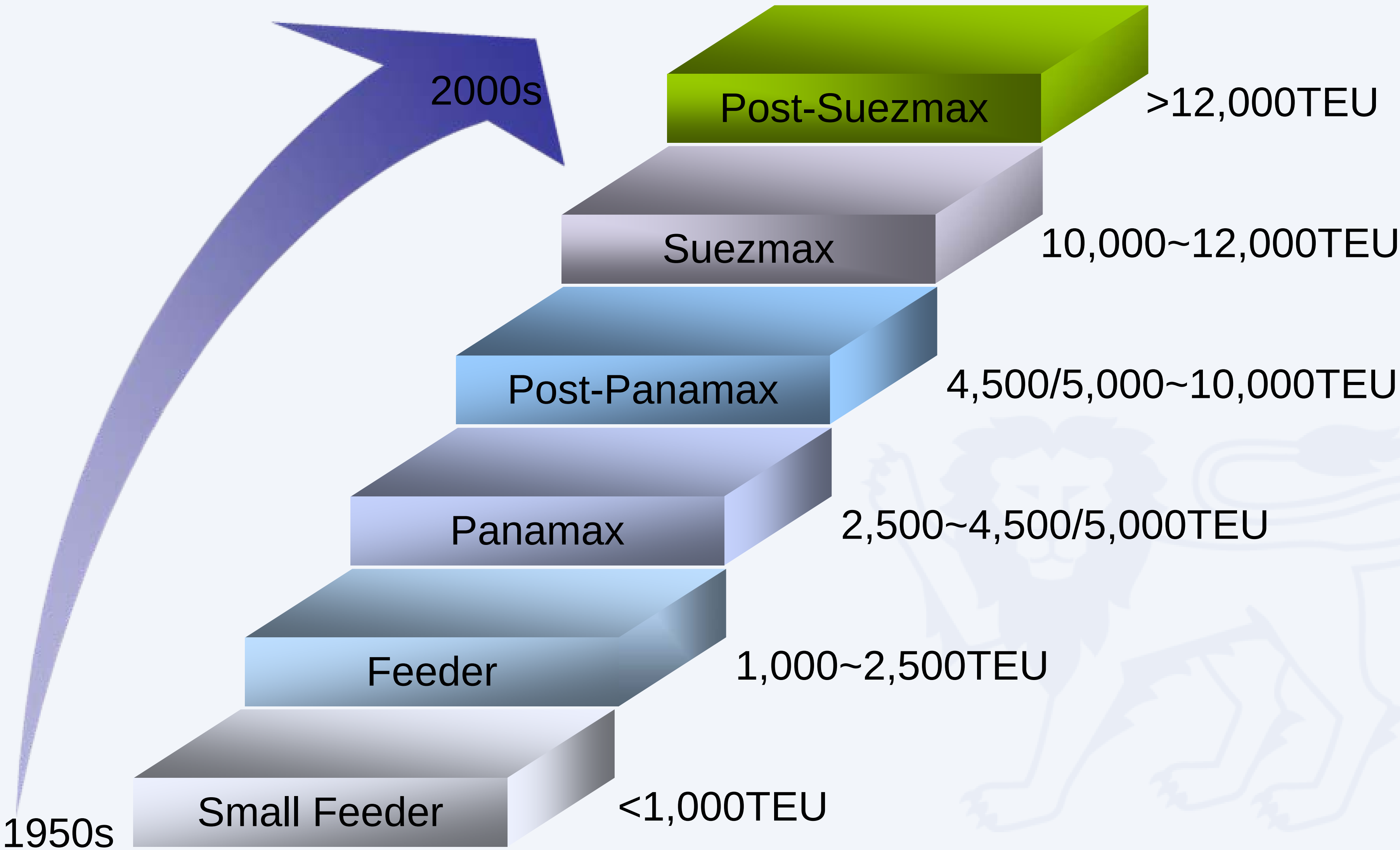


Source: Review of Maritime Transport, various issues

Introduction-Liner Shipping

- ❖ Provide a shipping service at a fixed and a regular schedule
- ❖ Sail on the fixed itineraries or routes
- ❖ Shipping cargoes from all comers and sail whether filled or not

Introduction- Containership Sizes



Introduction – Containership Fleet

Long-term Trends in the Cellular Containership Fleet for Selected Years

Year	Number of ships	Total capacity (TEU)	Average capacity (TEU)
2000	2,433	4,297,874	1,766
2001	2,595	4,734,079	1,824
2002	2,755	5,356,650	1,944
2003	2,890	5,896,154	2,044
2004	3,054	6,437,218	2,108
2005	3,206	7,165,352	2,235
2006	3,494	8,120,465	2,324
2007	3,904	9,436,377	2,417

Source: Compiled by the UNCTAD secretariat on the basis of data supplied by Lloyd's Register-Fairplay.

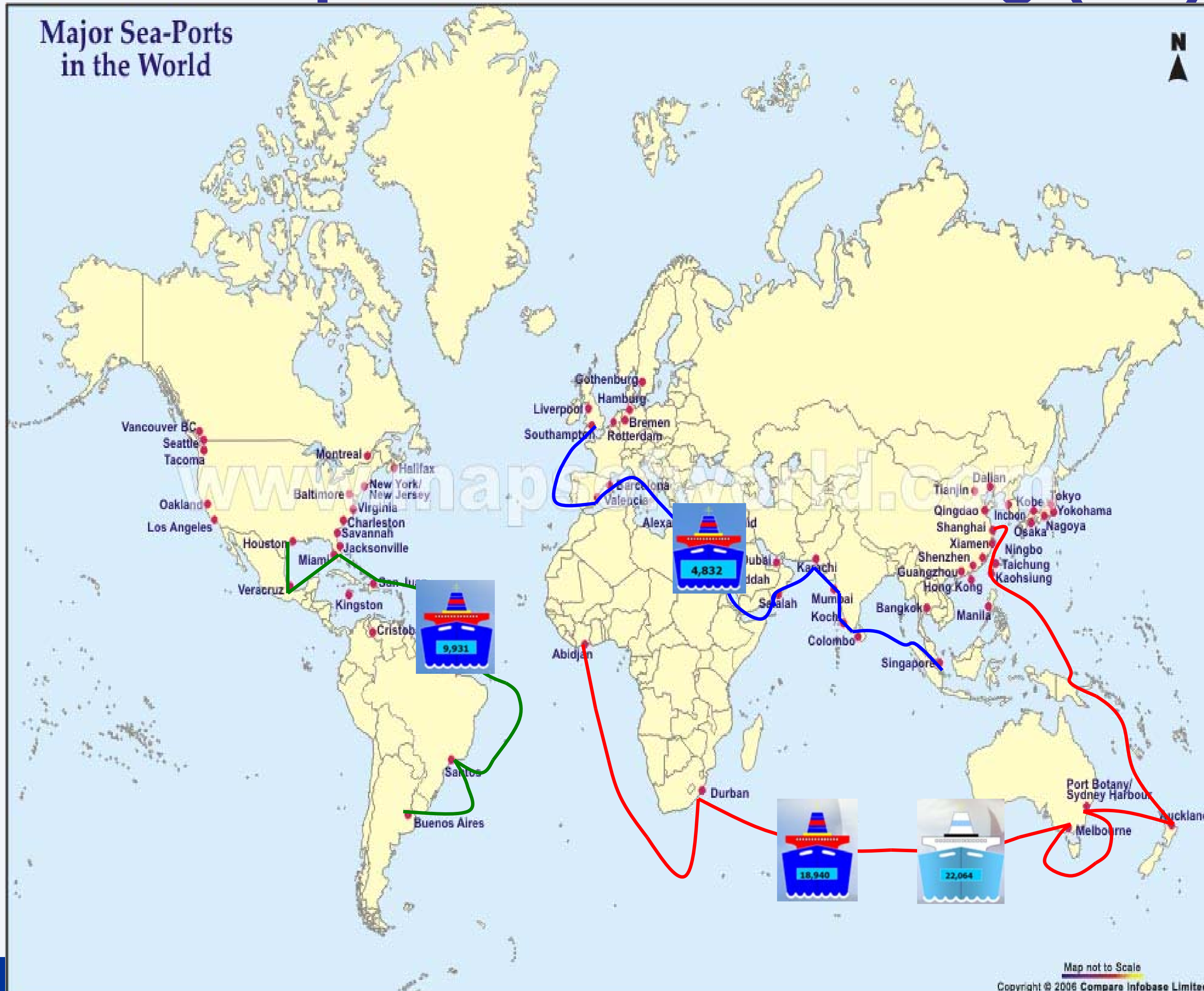
Introduction – Optimal Fleet Planning (1/2)

- Strategic planning
  - Ship types
  - number of ships
  - Time to purchase, charter-in or charter-out a ship

- Tactical planning
  - Assignment of ships to routes
  - Frequency on each route

- Operational planning
  - Cargo booking

Introduction – Optimal Fleet Planning (2/2)



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Literature Review-Representative Papers

Author	Year	Model	Level	Problem	Algorithm
T.A.J. Nicholson and R.D. Pullen	1971	DP	strategic	How to sale and replace the ships over years	Common DP algorithm
N.A. Papadakis and A.N Perakis	1989	NLP	tactical	How to assign the ships on routes	Predetermined speed then use the common LP algorithm
A.N Perakis and D. I. Jarmillo	1991	LP	tactical	How to assign the ships on routes	LINDO software
B.J. Powell and A.N. Perakis	1997	IP	tactical	How to assign the ships on routes	AMPL and OSL software
X.L. Xie and T.F. Wang	2000	DP	Mixed strategic and tactical	How to plan the fleet (size and mix), only adding new ships	Common DP algorithm
J.S.K. Ang, C.X. Cao and H.Q. Ye	2007	DP	operational	Which cargo to be carried and arrangement of the shipping schedule	Heuristic algorithm

Study Objectives

1. Propose a dynamic programming model mixed strategic planning level (for long term planning) and tactical planning level (for short term planning)
2. When, which and how many ships (charter-in, charter-out, sell or purchase) are needed each year
3. How to assign ships to routes
4. How to determine the optimal ships (speed)

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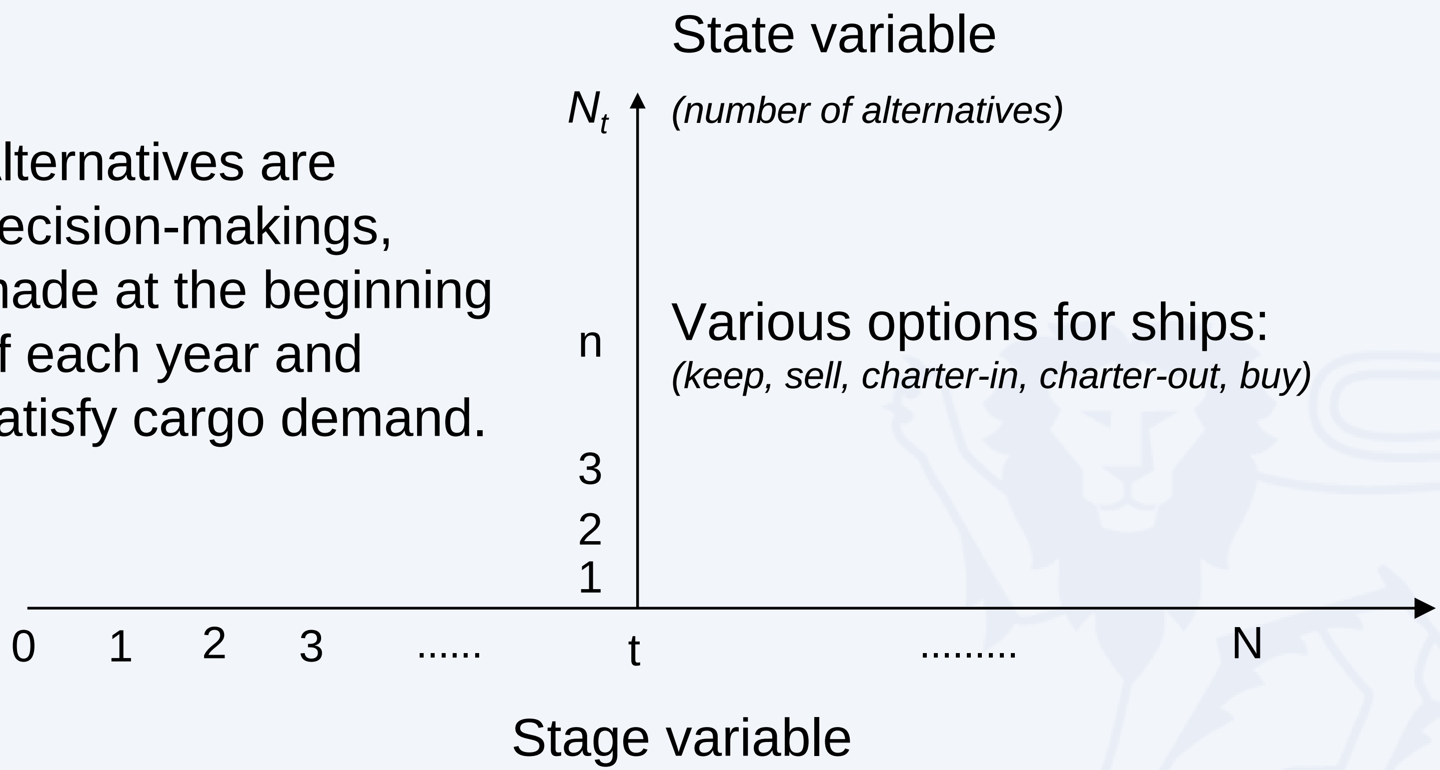
Dynamic Programming Model

Assumption

1. Port to port cargo demand is known
2. Possible routes are fixed in planning horizon
3. Waiting time from booking a new containerships to being finished is ignored
4. Decision of fleet size is made at the beginning of each year
5. Period for charter-out/in ship is one year every time

Dynamic Programming Model (cont'd)

Alternatives are decision-makings, made at the beginning of each year and satisfy cargo demand.



Dynamic Programming Model (cont'd)

$\Omega^0_{t,n}$	The old ships owned by a company in alternative n at beginning of year t
$\Omega^1_{t,n}$	The available ships in lease market in alternative n at beginning of year t
$\Omega^2_{t,n}$	The available new ships in alternative n at beginning of year t
$\Re_{t,n}$	The ships served in alternative n at beginning of year t
$\Psi^0_{t,n}$	The old own ships served in alternative n at beginning of year t
$\Psi^1_{t,n}$	The charter-in ships served in alternative n at beginning of year t
$\Psi^2_{t,n}$	The new purchased ships served in alternative n at beginning of year t
$\Psi^3_{t,n}$	The ships chartered-out in alternative n at beginning of year t
$\Psi^4_{t,n}$	The ships sold in alternative n at beginning of year t

Dynamic Programming Model (cont'd)

$$\Re_{t,n} = \Psi^0_{t,n} \bigcup \Psi^1_{t,n} \bigcup \Psi^2_{t,n}$$

$$\Psi^0_{t,n} \subseteq \Omega^0_{t,n}$$

$$\Psi^1_{t,n} \subseteq \Omega^1_{t,n}$$

$$\Psi^2_{t,n} \subseteq \Omega^2_{t,n}$$

$$\Longrightarrow$$

$$\Re_{t,n} \subseteq \Omega^0_{t,n} \bigcup \Omega^1_{t,n} \bigcup \Omega^2_{t,n}$$

$$\Omega^0_{t+1,n} = \Psi^0_{t,n} \bigcup \Psi^2_{t,n} \bigcup \Psi^3_{t,n}$$

$$\Omega^0_{t,n} = \Psi^0_{t,n} \bigcup \Psi^3_{t,n} \bigcup \Psi^4_{t,n}$$

Dynamic Programming Model (cont'd)

Capacity Estimation (Network)

speed

For one ship, annual capacity (tons/year): $\alpha_k x_k + \beta_k$

For one fleet, annual capacity: $\sum_{k \in \Re_{t,n}} (\alpha_k x_k + \beta_k)$

On each route, annual capacity: $\sum_{k \in \Re_{t,n}} (\alpha_k x_k + \beta_k) z_{kr}$ $\longrightarrow$ $z_{kr} = \begin{cases} 1 & \text{Ship k is on route r} \\ 0 & \text{otherwise} \end{cases}$

Resource: *Benford, H.(1981), A simple approach to fleet deployment, Maritime Policy and Management, Vol.8,N0.4,pp.223-228*

Dynamic Programming Model (cont’d)

Net profit=Revenue - Cost

Revenue: shipping cargoes + charter-out/sell ships

Cost: shipping cargoes + charter-in/buy ships + maintain ships

Dynamic Programming Model (cont'd)

Revenue :

The cargo from port i to port j on route r (tons)

Shipping cargoes: $$\mathrm{R}_{\mathrm{t,n}}^{\mathrm{shipping}} = \sum_{r=1}^{R} \sum_{i,j=1}^{I_r} q_{ij}^{r} \times p_{ij}^{r}$$

Freight rate ($/ton)

Charter-out/sell ships: $$\mathrm{R}_{t,n}^{\text{charter-out/sell}} = \sum_{k \in \Psi_{t,n}^{3} \cup \Psi_{t,n}^{4}} r_{t,n}^{ki}$$

Age i at the beginning of year t

The set of ships are chartered-out/sold

Dynamic Programming Model (cont'd)

Cost=Operating Cost + Investment + Maintenance Cost

Operating cost for one ton cargo carried by ship k: $\gamma_k x_k^2 + \delta_k x_k + \varepsilon_k$

Operating cost carried for cargo carried by ship k:

$$\sum_{k \in \Re_{t,n}} \left(D_k^t\right)\left(\gamma_k x_k^2 + \delta_k x_k + \varepsilon_k\right)$$

Resource: *Benford, H.(1981), A simple approach to fleet deployment, Maritime Policy and Management, Vol.8,N0.4,pp.223-228*

Dynamic Programming Model (cont'd)

Investment: $\sum_{k \in \Psi^1_{t,n}} c^k_{t,n} + \sum_{k \in \Psi^2_{t,n}} p^k_{t,n}$

Maintenance cost: $\sum_{k \in \Re_{t,n}} m^k_{t,n}$

Cost: $C_{t,n} = \sum_{k \in \Re_{t,n}} \left(D^t_k\right)\left(\gamma_k x^2_k + \delta_k x_k + \varepsilon_k\right) z_{kr} + \sum_{k \in \Psi^1_{t,n}} c^k_{t,n} + \sum_{k \in \Psi^2_{t,n}} p^k_{t,n} + \sum_{k \in \Re_{t,n}} m^k_{t,n}$

Dynamic Programming Model (cont'd)

Feasible Alternatives

For each feasible alternative *n* in year *t*, decide the optimal speed and routing s.t.

$$\mathrm{NP}_{t,n} = \max\left(\mathrm{R}_{t,n}^{\text{shipping}} + \mathrm{R}_{t,n}^{\text{charter-out/sell}} - \left(\sum_{k\in\Re_{t,n}} \left(D_k^t\right)\left(\gamma_k x_k^2 + \delta_k x_k + \varepsilon_k\right) z_{kr} + \sum_{k\in\Psi_{t,n}^1} c_{t,n}^k + \sum_{k\in\Psi_{t,n}^2} p_{t,n}^k + \sum_{k\in\Re_{t,n}} m_{t,n}^k \right)\right)$$

constraints

$$\sum_{k\in\Re_{t,n}} \left(\alpha_k x_k + \beta_k\right) z_{kr} \geq D_t^r$$ → Demand in year t on route r

$$\Re_{t,n} = \Psi_{t,n}^0 \cup \Psi_{t,n}^1 \cup \Psi_{t,n}^2$$ → Ships in service: own, charter-in, purchased

Dynamic Programming Model (cont'd)

Bellman equation:

$$TNP_{t,n} = NP_{t,n} + \max_{m=1,\ldots,N^{n}_{t+1}} TNP_{t+1,m}$$

Total net profit from year *t* to year *N*

The number of alternatives *s.t.*

$$\Psi^{0}_{t+1,m} \subseteq \Psi^{0}_{t,n} \cup \Psi^{2}_{t,n} \cup \Psi^{3}_{t,n}$$

Objective: $\max_{n=1,\ldots,N_0} TNP_{0,n}$

Constraints: $\Omega^{0}_{t+1,n} = \Psi^{0}_{t,n} \cup \Psi^{2}_{t,n} \cup \Psi^{3}_{t,n}$

$$\Omega^{0}_{t,n} = \Psi^{0}_{t,n} \cup \Psi^{3}_{t,n} \cup \Psi^{4}_{t,n}$$

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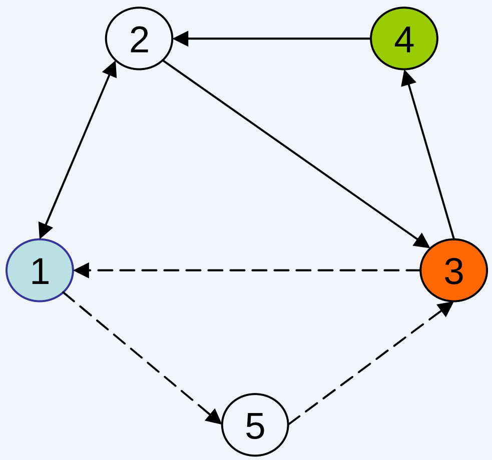
Numerical Example



Route 1: 1 → 2 → 3 → 4 → 2 → 1 2 weeks

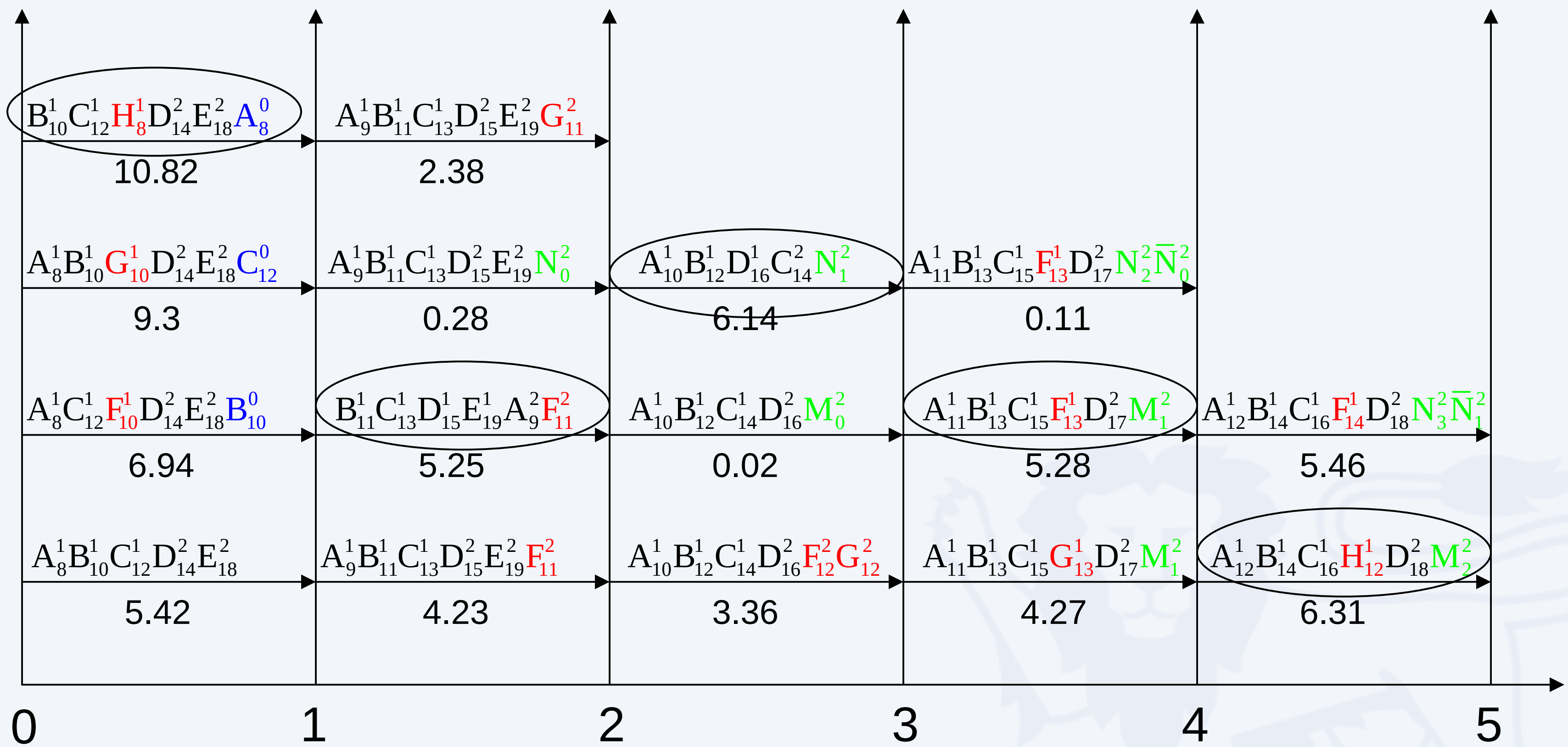
Route 2: 1 - - → 5 - - → 3 - - → 1 1 week

Ship	Capacity (TEU)	Age	Shipping cost (M$/trip) Route 1	Shipping cost (M$/trip) Route 2	Lease cost (M$/year)	Maintenance cost (M$/year)
A	5000	8	0.20	0.10	13	0.25+0.01y
B	4500	10	0.18	0.09	9	0.20+0.01y
C	4000	12	0.16	0.08	8	0.20+0.01y
D	2000	14	0.06	0.04	4	0.10+0.01y
E	1500	18	0.06	0.03	3	0.15+0.01y
F	3500	10	0.15	0.06	6	0.15+0.01y
G	3000	10	0.14	0.05	5	0.10+0.01y
H	4500	8	0.18	0.09	9	0.20+0.01y
M	3000	0	0.12	0.05	6	0.01+0.01y
N	2000	0	0.06	0.03	5	0.01+0.01y

	Demand (M Tons)	1 → 2	2 → 3	3 → 4	4 → 2	2 → 1
Route 1	Year 1	0.25	0.20	0.20	0.15	0.30
	Year 2	0.28	0.21	0.22	0.20	0.25
	Year 3	0.30	0.25	0.25	0.25	0.45
	Year 4	0.32	0.25	0.28	0.30	0.42
	Year 5	0.35	0.30	0.25	0.35	0.55

	Demand	1 - - → 5	5 - - → 3	3 - - → 1
Route 2	Year 1	0.10	0.12	0.08
	Year 2	0.18	0.16	0.11
	Year 3	0.20	0.19	0.15
	Year 4	0.24	0.22	0.18
	Year 5	0.30	0.28	0.27

Numerical Example (cont'd)



Note: black: currently own ships; green: new purchased ships; red: charter-in ships; blue: charter-out ships

Conclusion

Long-term planning: DP

Short-term planning: MINLP

- Shipping capacity estimation
- Size of fleet
- Deployment
- Speed

Algorithm

Thank You !